DISCRETE COSINE TRANSFORM

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Agenda

- DCT
- Algorithms for conventional DCT
- Example
- Directional DCT

DCT

- A technique for converting signal into elementary frequency components

- Why we need compression?
  The need for sufficient storage space, large transmission bandwidth, and long transmission time for image, audio, and video data

Principles behind compression

- Redundancy reduction
  Aims at removing duplication from the signal source

- Irrelevancy reduction
  It omits parts of the signal that will not be noticed by the signal receiver.

Coding

- Predictive Coding - In predictive coding, information already sent or available is used to predict future values, and the difference is coded.
- Transform Coding - Transforms the image from its spatial domain representation to a different type of representation using some wellknown transform and then codes the transformed values (coefficients)

Continued..

Transform coding relies on the premise that pixels in an image exhibit a certain level of correlation with their neighboring pixels

Consequently, these correlations can be exploited to predict the value of a pixel from its respective neighbours.

One-Dimensional Discrete Cosine Transform

The DCT can be written as the product of a vector (the input list) and the n x n orthogonal matrix whose rows are the basis Vectors.

We can find that the matrix is orthogonal And each basis vector corresponds to a sinusoid of a certain frequency. The general equation for a 1D (*N* data items)

DCT is defined by the following equation:

$$F(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \Lambda(i).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] f(i)$$

where ,

$$\Lambda(i) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

This equation expresses *F* as a linear combination of the basis vectors. The coefficients are the elements of the inverse transform, which may be regarded as reflecting the amount of each frequency present in the input *F.*

The one-dimensional DCT is useful in processing one-dimensional signals such as speech waveforms. For analysis of two-dimensional (2D) signals such as images, we need a 2D version of the DCT.

The Two-Dimensional DCT

1D DCT is applied to each row of *F* and then to each column of the result.The general equation for a 2D (*N* by *M* image) DCT is defined by the following equation:

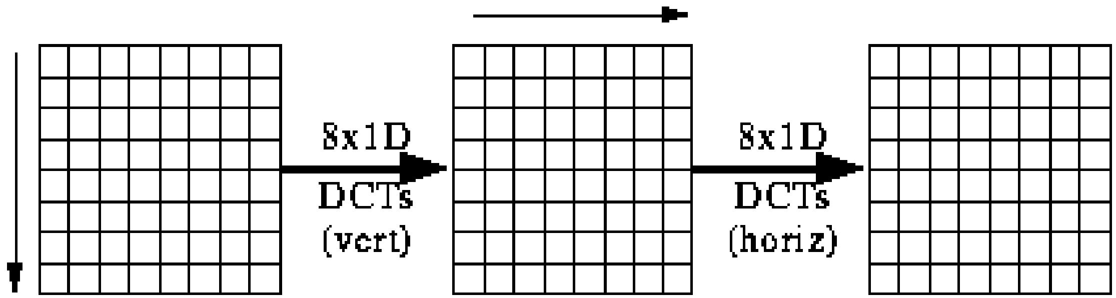
$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i).\Lambda(j).cos\left[\frac{\pi.u}{2.N}(2i+1)\right] cos\left[\frac{\pi.v}{2.M}(2j+1)\right].f(i,j)$$

and the corresponding *inverse* 2D DCT transform is simple $F^{-1}(u,v)$,

where

$$\Lambda(\xi) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for} \xi = 0 \\ 1 & \text{otherwise} \end{cases}$$

The equations are given by:



Since the 2D DCT can be computed by applying 1D transforms separately to the rows and columns, we say that the 2D DCT is *separable* in the two dimensions.

A Typical Lossy Signal/Image Encoder

- Source Encoder (or Linear Transformer) used is Discrete Cosine Transform (DCT)
- Quantizer -A quantizer simply reduces the number of bits needed to store the transformed coefficients by reducing the precision of those values. Quantization can be performed on each individual coefficient, which is known as Scalar Quantization (SQ).

Continued ...

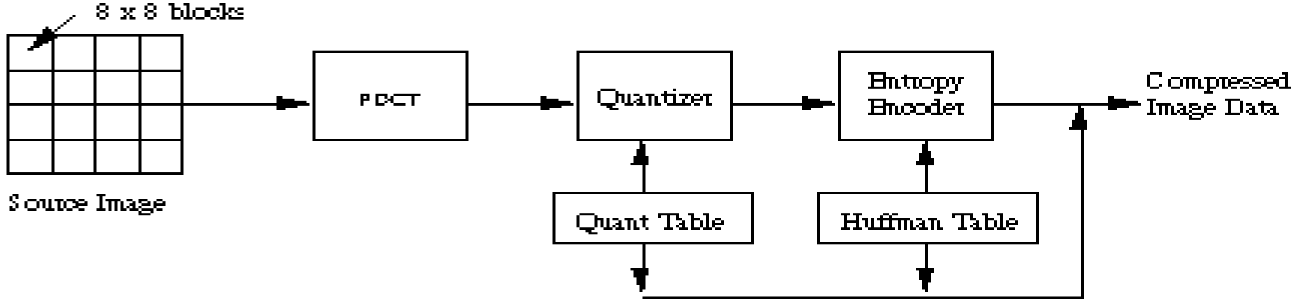
- Entropy Encoder - An entropy encoder further compresses the quantized values losslessly to give better overall compression by accurately determine the probabilities for each quantized value and produces an appropriate code based on it, so that the resultant output code stream will be smaller than the input stream

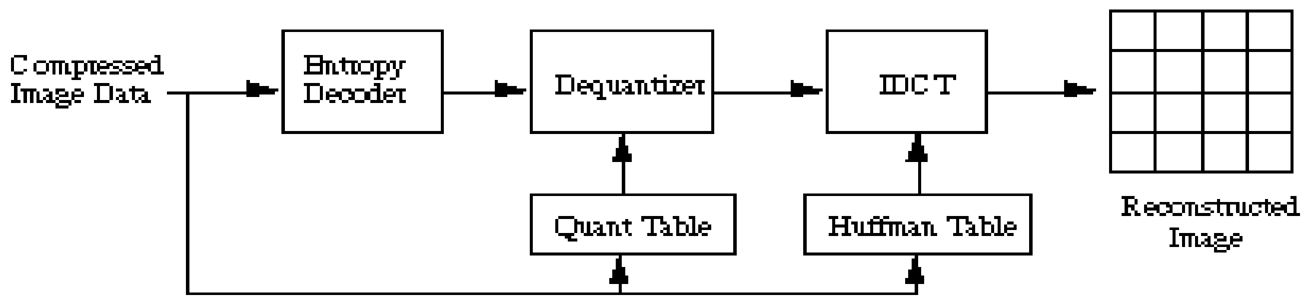
eg. Huffman encoder and the arithmetic encoder.

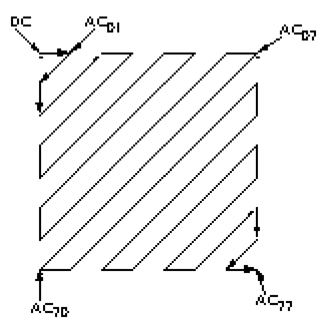
DCT-based are either compressed entirely one at a time, or are compressed by alternately interleaving 8x8 sample blocks from each in turn. For a typical 8x8 sample block from a typical source image, most of the spatial frequencies have zero or near-zero amplitude and need not be encoded. DCT introduces no loss to the source image samples.

DCT merely transforms them to a domain in which they can be more efficiently encoded. The DC coefficient, which contains a significant fraction of the total image energy, is differentially encoded. Entropy Coding (EC) achieves additional compression losslessly by encoding the quantized DCT coefficients more compactly based on their statistical characteristics.

While the DCT-based image coders perform very well at moderate bit rates, at higher compression ratios, image quality degrades because of the artifacts resulting from the block-based DCT scheme.


8 x 8 blocks
FDCT
Quantizer
Entropy Encoder
Compressed Image Data
Source Image
Quant Table
Huffman Table


Compressed Image Data
Entropy Decoder
Dequantizer
IDCT
Quant Table
Huffman Table
Reconstructed Image


DC
AC01
AC07
AC70
AC77

Sample 8*8 block

52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	114	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

Level Shifted by -128

-76	-73	-67	-62	-58	-67	-64	-55
-65	-69	-62	-38	-19	-43	-59	-56
-66	-69	-60	-15	16	-24	-62	-55
-65	-70	-57	-6	26	-22	-58	-59
-61	-67	-60	-24	-2	-40	-60	-58
-49	-63	-68	-58	-51	-65	-70	-53
-43	-57	-64	-69	-73	-67	-63	-45
-41	-49	-59	-60	-63	-52	-50	-34

DCT Co-efficients (After transform)

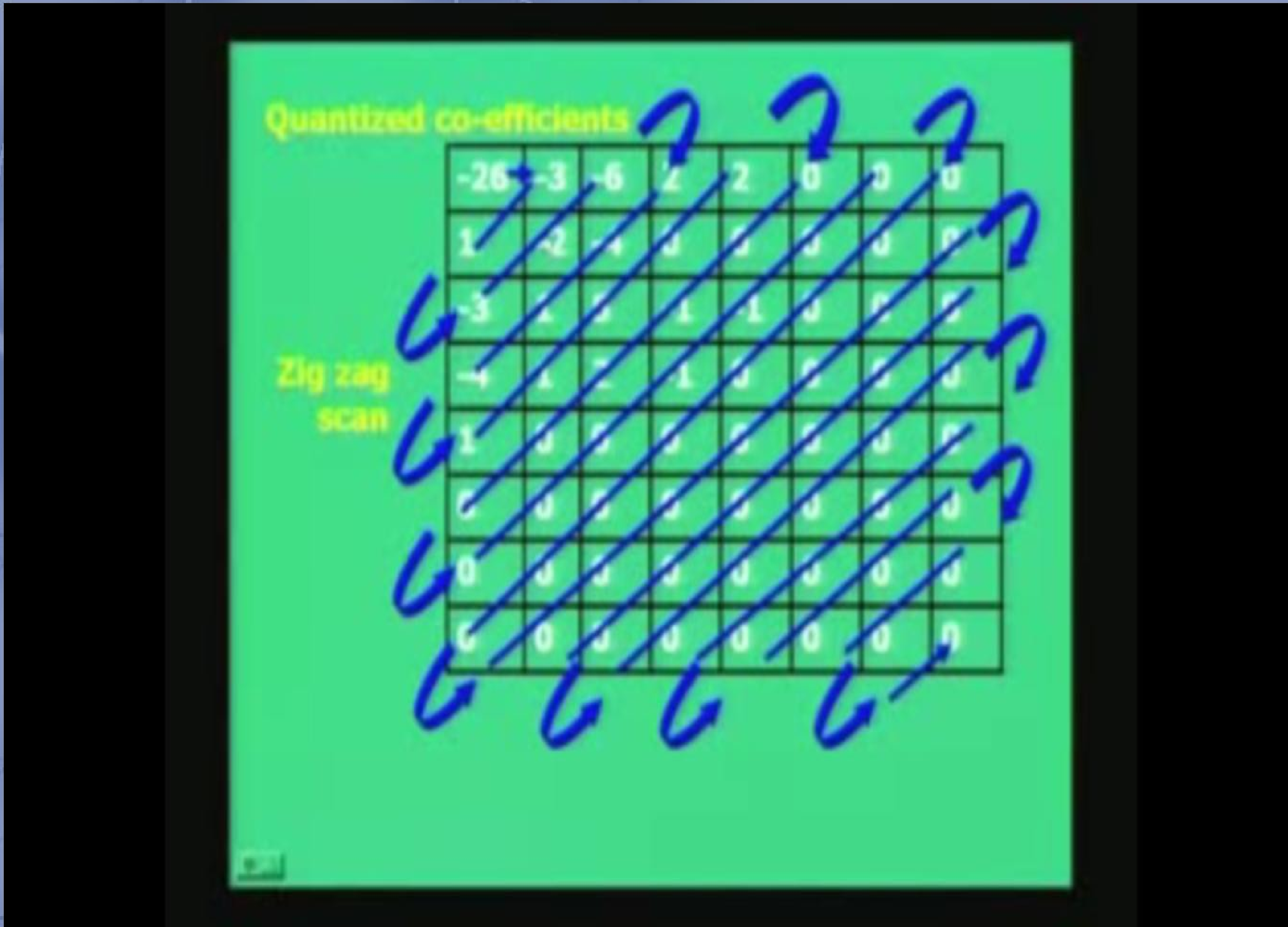
DC co-efficient

-415	-29	-62	25	55	-20	-1	3
7	-21	-62	9	11	-7	-6	6
-46	8	77	-25	-30	10	7	-5
-50	13	35	-15	-9	6	0	3
11	-8	-13	-2	-1	1	-4	1
-10	1	3	-3	-1	0	2	-1
-4	-1	2	-1	2	-3	1	-2
-1	-1	-1	-2	-1	-1	0	-1

Quantization table

Divided by

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99


Quantized co-efficients
Zig zag
scan
-26 -3 -6 2 2 0 0 0

- Zig Zag scan is done so that the coefficients are in order of increasing frequency.
- The higher frequency coefficients are more likely to be 0 after quantization.
- This improves the compression of run-length encoding.

The coded data is entropy decoded in the decoder

-26	-3	-6	2	2	0	0	0
1	-2	-4	0	0	0	0	0
-3	1	5	-1	-1	0	0	0
-4	1	2	-1	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Quantization table

Multiplied by:

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Denormalized co-efficients

-416	-33	-60	32	48	0	0	0
12	-24	-56	0	0	0	0	0
-42	13	80	-24	-40	0	0	0
-56	17	44	-29	0	0	0	0
18	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

After Inverse DCT

-70	-64	-61	-64	-69	-66	-58	-50
-72	-73	-61	-39	-30	-40	-54	-59
-68	-78	-58	-9	13	-12	-48	-64
-59	-77	-57	0	22	-13	-51	-60
-54	-75	-64	-23	-13	-44	-63	-56
-52	-71	-72	-54	-54	-71	-71	-54
-45	-59	-70	-68	-67	-67	-61	-50
-35	-47	-61	-66	-60	-48	-44	-44

After level shifting (reconstructed subimage)

58	64	67	59	62	68	70	78
56	55	67	89	98	88	74	69
60	50	70	119	141	116	80	64
69	51	71	128	149	115	77	68
74	53	64	105	115	84	65	72
76	57	56	74	75	57	57	74
83	69	59	60	61	61	67	78
93	81	67	62	69	80	84	84

Original subimage

52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	114	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

Difference between original and reconstructed subimage

-6	-9	-6	2	11	-1	-6	-5
7	4	-1	1	11	-3	-5	3
2	9	-2	-6	-3	-12	-14	9
-6	7	0	-4	-5	-9	-7	1
-7	8	4	-1	11	4	3	-2
3	8	4	-4	2	11	1	1
2	2	5	-1	-6	0	-2	5
-6	-2	2	6	-4	-4	-6	10

Properties of DCT

- Decorrelation - The principle advantage of image transformation is the removal of redundancy between neighbouring pixels. This leads to uncorrelated transform coefficients which can be encoded independently.

- Energy Compaction - DCT exhibits excellent energy compaction for highly correlated images. The uncorrelated image has its energy spread out, whereas the energy of the correlated image is packed into the low frequency region.

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- Orthogonality - IDCT basis functions are orthogonal . Thus, the inverse transformation matrix of A is equal to its transpose i.e. invA= A'.
- Separability – Perform DCT operation in any of the direction first and then apply on second direction, coefficient will not change

Advantages and Disadvantages

- The DCT does a better job of concentrating energy into lower order coefficients than does the DFT for image data

- The DCT is purely real, the DFT is complex.

- Assuming a periodic input, the magnitude of the DFT coefficients is spatially invariant . This is not true for the DCT

Directional Discrete Cosine Transforms

- Nearly all block based transform techniques uses 1D DCT or 2D DCT i.e Conventional DCT

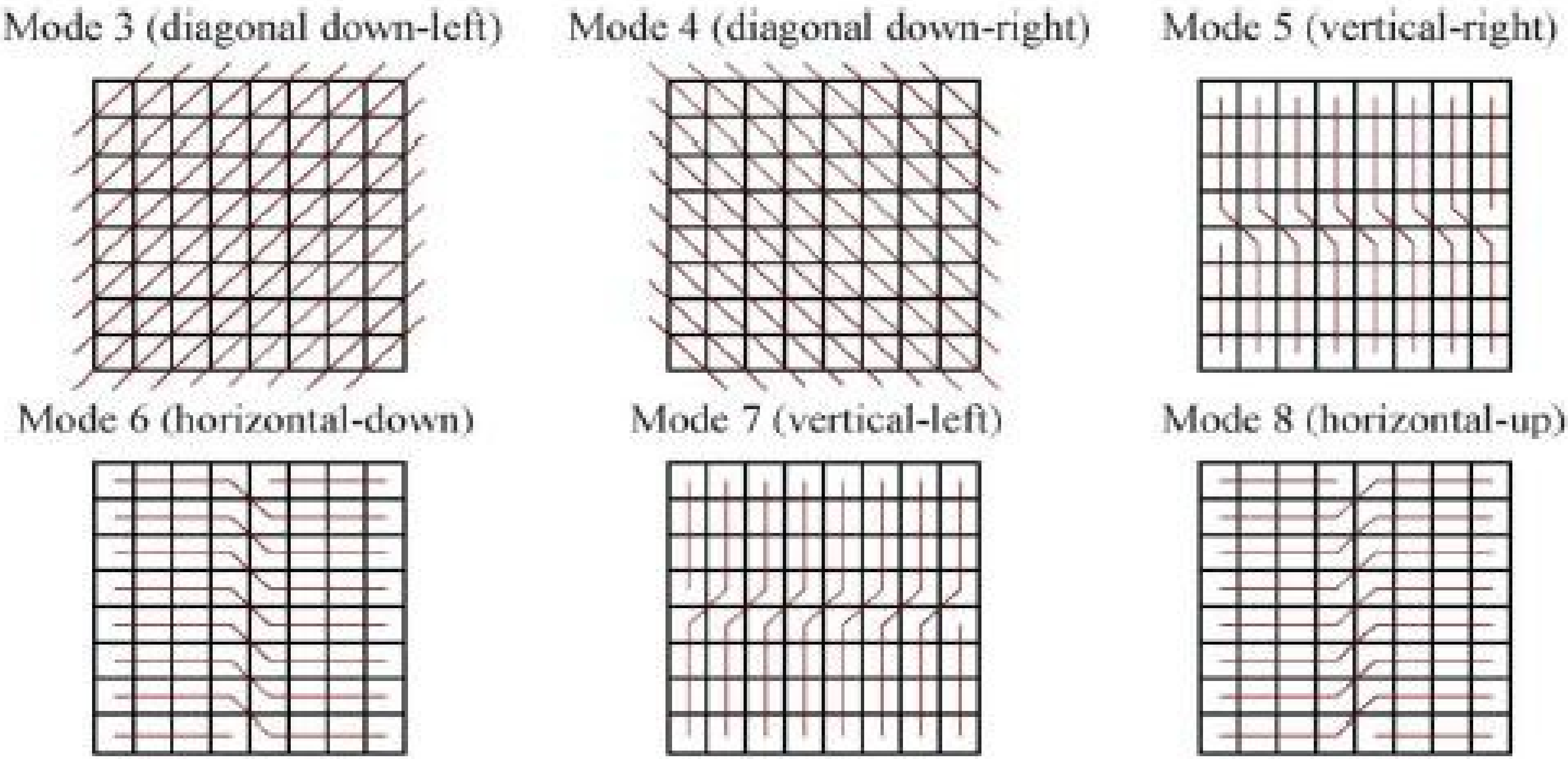
- But Image blocks may contain edges in direction other than horizontal and vertical

- Inorder to improve coding performance we use directional DCT in above cases

Continued..

- The conventional N*N 2-D DCT is always implemented separately by two N point 1-D DCTs
- 2D conventional DCT may cause some defects when it is applied to an image block in which other directional edges dominate
- If we apply 1D DCT then we get some nonzero coefficient that are not aligned after applying second 1D DCT we may produce more nonzero coefficients.

Directional DCT



Directional DCT for the Diagonal Down-Left Mode

Algorithm

1. The first 1-D DCT will be performed along the diagonal down-left direction. All of the coefficients are expressed in group of column vector
2. Second 1-D DCT is applied to each row
3. Then push the coefficients to horizontally left
4. Perform zigzag scanning to convert 2D coefficient block into 1D sequence

Problems

- Mean Wieghting defect

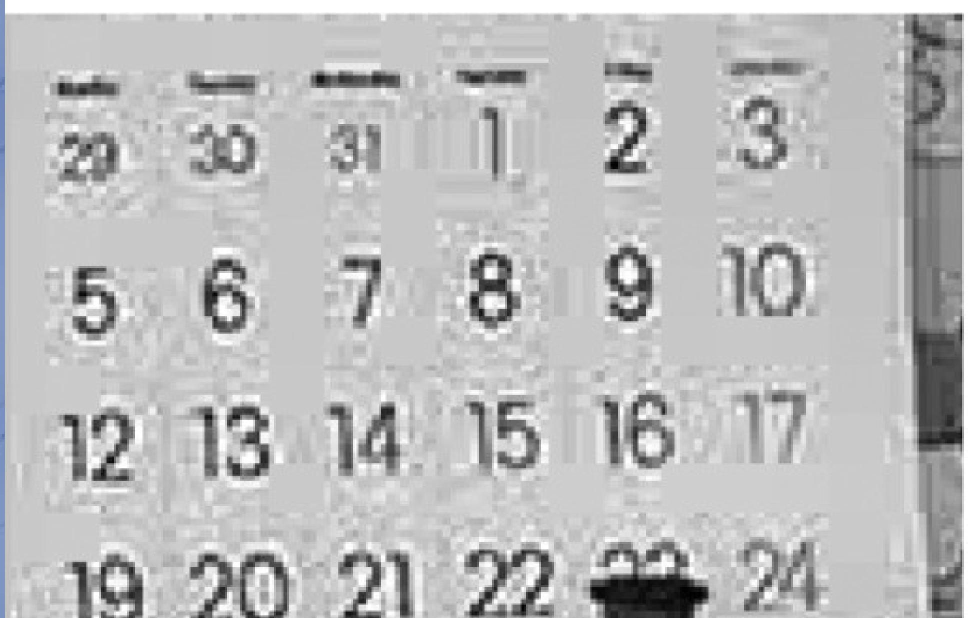
- Solution to above step using diagonal length will produce noise wieghting defect

Solution

- Quantized mean value of image block is subtracted from image block
- Apply diagonal left DCT
- Apply second 1D DCT horizontally and coefficients are pushed to left

(a)

(b)


29 30 31 1 2 3
5 6 7 8 9 10
12 13 14 15 16 17
19 20 21 22 24

(c)


29 30 31 1 2 3
5 6 7 8 9 10
12 13 14 15 16 17
19 20 21 22 24

(d)

Refrences

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2.http://www.youtube.com/watch?v=hgr5O0du-sg
3.http://wisnet.seecs.edu.pk/publications/tech_reports/DCT_TR802.pdf
4. Directional Discrete Cosine Transforms—A New Framework for Image Coding by Bing Zeng, *Member, IEEE*, and Jingjing Fu http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=4449470&isnumber=4479597

End

Questions??